



Decimals (part 1)

Theory and examples

Learning Skills

Introduction:

A decimal involves a number system based on 10. To understand decimals you first need to understand place values which can be found in our [number](#) document. An understanding of decimals and how to add, subtract, multiply and divide is a foundation for work in primary mathematics. Explanations of some of the terms that have been used in this document can be found in the [glossary](#) on our website. [Part 2](#) of this document contains exercises to practise

This sheet will teach you to:

- Recognise place value for decimals 1
- Write decimals in expanded form 2
- Compare decimals 3
- Multiply by a power of ten 3
- Divide by a power of ten 4
- Add or subtract decimals 5
- Multiply decimals 5
- Divide decimals when the divisor is a whole number 6
- Divide decimals when the divisor is not a whole number 7
- Convert other fractions to decimals 8
- Round off 9

1. Place value for decimals

We use a decimal place to represent numbers greater than or less than one by placing them to the left or right of the decimal place. The following is a reminder of place values including past the decimal point.

Tens	Units	Decimal point	Tenths or $\frac{1}{10}$'s	Hundredths or $\frac{1}{100}$'s	Thousandths or $\frac{1}{1000}$'s
3	6	.	5	2	9

Notes

- The number to the left of the decimal place is a whole number
- The first digit on the right of the decimal place represents $\frac{1}{10}$
- As we move to the left each position is 10 times bigger
- As we move to the right each position is 10 times smaller

$\frac{7}{10}$ Tenths fill the first place value after the decimal point so the decimal form is 0.7

Units	Dec. point	Tenths
0	•	7

$\frac{23}{100}$ Hundredths fill the first two decimal places so the decimal form is 0.23

Units	Dec. point	Tenths	Hundredths
0	•	2	3

$\frac{3}{100}$ The decimal form is 0.03

Units	Dec. point	Tenths	Hundredths
0	•	0	3

Note

The number of zeros in the denominator matches the number of decimal places the decimal takes.

$\frac{478}{1000}$ The decimal form is 0.478

Units	Dec. point	Tenths	Hundredths	Thousandths
0	•	4	7	8

$\frac{29}{1000}$ The decimal form is 0.029

Units	Dec. point	Tenths	Hundredths	Thousandths
0	•	0	2	9

2. Expanded form of decimals

Decimals, like whole numbers can be written in expanded form

Examples

a) 0.23

Units 1's	Dec. point	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
0	•	2	3	

Expanded form: 2 tenths + 3 hundredths

$$\text{Or } 2 \times \frac{1}{10} + 3 \times \frac{1}{100}$$

b) 2.145

Units 1's	Dec. point	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
2	•	1	4	5

Expanded form: 2 units + 1 tenth + 4 hundredths + 5 thousandths

$$\text{Or } 2 \times 1 + 1 \times \frac{1}{10} + 4 \times \frac{1}{100} + 5 \times \frac{1}{1000}$$

3. Comparing decimals

To compare decimals give each decimal the same number of decimal places by filling the decimal places with 0's, then compare each place starting with the tenths.

Examples

a) Write in ascending order: 0.3, 0.33 and 0.303

Units 1's	Dec. point	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
0	•	3	0	0
0	•	3	3	0
0	•	3	0	3

After giving all the decimals 3 decimal places it is easier to see that 0.300 or 0.3 is the smallest, 0.303 is the next and 0.330 or 0.33 is the biggest decimal.

4. Multiplying by a power of ten

To multiply by a power of ten, we move the decimal point to the right the same number of places as the number of noughts in the power of ten. (You may need to add some spare noughts)

Examples

a) 2.56×10

$$2 \cdot 5 \cdot 6 = 25.6$$

Tens	Units	Dec. point	Tenths	Hundredths
	2	•	5	6
2	5	•	6	

b) 0.83×10

$$0 \cdot 8 \cdot 3 = 8.3$$

Tens	Units	Dec. point	Tenths	Hundredths
	0	•	8	3
	8	•	3	

c) 35.6×100 3 5 • 6 0 • = 3560

(a 0 needs to be added after the 6 to fill in the place value)

		3	5	•	6
3	5	6	0	•	0

d) 0.62×100 0 • 6 2 • = 62

	0	•	6	2
6	2	•	0	

e) 34×1000

The decimal point is not showing here but we know that it is after the 4. Move the decimal point 3 places to the right or add 3 noughts to the whole number 34

3 4 • 0 0 0 • = 34000

	0	•	8	3
	8	•	3	

5. Dividing by a power of ten

To divide by a power of ten, we move the decimal point to the left the same number of places as the number of noughts in the power of ten. (You may need to add some spare noughts)

Examples

a) $6325.4 \div 10$ 6 3 2 • 5 • 4 = 632.54

Thousands	Hundreds	Tens	Units		Tenths	hundredths
6	3	2	5	•	4	
	6	3	2	•	5	4

b) $823.5 \div 100$ 8 • 2 3 • 5 = 8.235

Hundreds	Tens	Units		Tenths	hundredths	Thousandths
8	2	3	•	5		
		8	•	2	3	5

c) $0.658 \div 100$ 0 • 0 0 • 6 5 8 = 0.00658

(two 0's need to be added before the 6 to fill in the place value)

0	•	6	5	8		
0	•	0	0	6	5	8

d) $13.8 \div 1000$

0 . 0 1 3 8 = 0.0138

1	3	.	8		
0	0	.	1	3	8

e) 34×1000

The decimal point is not showing here but we know that it is after the 4. Move the decimal point 3 places to the right or add 3 noughts to the whole number 34

3 4 . 0 0 0 = 34000

Ten thousandths	Thousandths	Hundreds	Tens	Units	Dec. point
			3	4	.
3	4	0	0	0	.

6. Adding or subtracting decimals

To add or subtract decimals align the decimal places and fill in any blank places with noughts then add or subtract place by place.

Examples

a) $25.68 + 5.279$

$$\begin{array}{r} 25.680 \\ + 5.279 \\ \hline 30.959 \end{array}$$

b) $181.92 - 53.43$

$$\begin{array}{r} 181.92 \\ - 53.43 \\ \hline 128.49 \end{array}$$

7. Multiplying decimals

To multiply decimals **multiply the digits then count the number of decimal places in the question and give the answer the same number of decimal places.**

Examples

a) 8×0.3

work out $8 \times 3 \rightarrow 24$

count the decimal places: 8 has 0 dp
0.3 has 1 dp

total dp = $0 + 1 = 1$

now give the answer 1 decimal place by putting a decimal point 1 place in from the end

$$= 2.4$$

Estimate: 0.3 is a little less than $\frac{1}{2}$ and $\frac{1}{2}$ of 8 is 4, so 2.4 looks OK.

b) 0.7×0.6

work out $7 \times 6 \rightarrow 42$

0.7 has 1 dp, 0.6 has 1 dp, total dp = 2 now give the answer 2 decimal place by putting a decimal point 2 places in from the end

$$= 0.42$$

Estimate: 0.6 is a little more than $\frac{1}{2}$ and $\frac{1}{2}$ of 0.7 is 0.35, so 0.42 looks OK.

c) 600×0.2

work out $600 \times 2 \rightarrow 6 \times 2(00) = 1200$

600 has 0 dp, 0.2 has 1 dp, total dp = 1

now give the answer 1 decimal place by putting a decimal point 1 place in from the end

$$= 120.0 \text{ or just } 120$$

Estimate: $600 \times 0.5 = 300$ so 120 looks OK.

d) 12.3×2.1

work out 123×21

$$\begin{array}{r} 123 \times \\ \underline{21} \\ 123 \\ \underline{2460} \\ 2583 \end{array}$$

12.3 has 1 dp, 2.1 has 1 dp, total dp = 2

now give the answer 2 decimal place by putting a decimal point 2 places in from the end

$$= 25.83$$

Estimate: 12×2 is 24. So our answer of 25.83 looks OK.

e) 0.5×0.0011

$5 \times 11 \rightarrow 55$

0.5 has 1 dp, 0.0011 has 4 dp, total dp = 5

now give the answer 5 decimal places

$$= 0.00055$$

Estimate: $\frac{1}{2}$ of $11/10000$ is $5.5/10000$.

8. Dividing decimals – divisor is a whole number

To divide decimals: when the number you are dividing by (divisor) is a whole number - Divide in decimal places. You may need to add some noughts to make more decimal places for the division.

Examples

a) $4.6 \div 2$

Note

That the decimal points need to line up

$$\begin{array}{r} 2 \cdot 3 \\ 2 \overline{)4 \cdot 6} \\ \hline \end{array} = 2.3$$

Estimate: 4 divided by 2 is 2 so 2.3 looks OK.

b) $26.45 \div 5$

$$\begin{array}{r} 5 \cdot 29 \\ 5 \overline{)26.45} \\ \hline \end{array} = 5.29$$

Estimate: 25 divided by 5 is 5 so 5.29 looks OK.

9. Dividing decimals – divisor is not a whole number

To divide decimals: If the divisor is not a whole number: multiply both numbers by a power of 10 to make the number you are dividing by a whole number

Examples

a) $2.4 \div 0.4$

make the 0.4 into a whole number by multiplying by 10

$$= \frac{2.4 \times 10}{0.4 \times 10} = \frac{24}{4}$$

$$\text{so } 2.4 \div 0.4 = 24 \div 4 = 6$$

Note

Remember that division means sharing into equal groups. So if 2.4 units of length is divided into 0.4 unit lengths, how many sections are there?

Sketch it and see

b) $0.8 \div 0.02$

make the 0.02 into a whole number by multiplying by 100

$$= \frac{0.8 \times 100}{0.02 \times 100} = \frac{80}{2}$$

$$\text{so } 0.8 \div 0.02 = 80 \div 2 = 40$$

Estimate: there are 50 lots of 0.02 shares to make 1 whole so 40 to make 0.8 looks OK.

c) $0.02 \div 0.8$

make the 0.8 into a whole number by multiplying by 10

$$= \frac{0.02 \times 10}{0.8 \times 10} = \frac{0.2}{8}$$

$$\text{so } 0.02 \div 0.8 = 0.2 \div 8$$

$$\begin{array}{r} 0.025 \\ 8 \overline{)0.200} \\ \hline \end{array}$$

$$= 0.025$$

Estimate: 0.8 is a much bigger portion than 0.02 so the number of shares will be a lot less than 1.

10. Converting other fractions to decimals

We can convert fractions to decimals by two methods:

Method 1

By forming an equivalent fraction with 10 or 100 in the denominator – this only works with some denominators, but is the easier method.

Method 2

Divide the numerator by the denominator (add two or three noughts after the decimal point so that you have something to divide into) – this works with all fractions.

Examples

a) $\frac{1}{2}$

$$\frac{1}{2} \Rightarrow \frac{1^{\times 5}}{2^{\times 5}} \Rightarrow \frac{5}{10} \Rightarrow 0.5$$

or divide $1 \div 2$

$$\begin{array}{r} 0.5 \\ 2 \overline{) 1.0} \end{array}$$

b) $\frac{9}{25}$

$$\frac{9}{25} \Rightarrow \frac{9^{\times 4}}{25^{\times 4}} \Rightarrow \frac{36}{100} \Rightarrow 0.36$$

or divide 9 by 25

$$\begin{array}{r} 0.36 \\ 25 \overline{) 9.00} \end{array}$$

c) $\frac{5}{8}$

This fraction cannot be made into a 1/10th or 1/100th so we need to use the division method.

divide 5 by 8

$$\begin{array}{r} 0.625 \\ 8 \overline{) 5.000} \end{array}$$

d) $\frac{3}{40}$

$$\begin{array}{r} 0.075 \\ 40 \overline{) 3.000} \end{array}$$

$$= 0.075$$

