



# Fractions (part 1)

## Theory and examples

### Learning Skills

### Introduction:

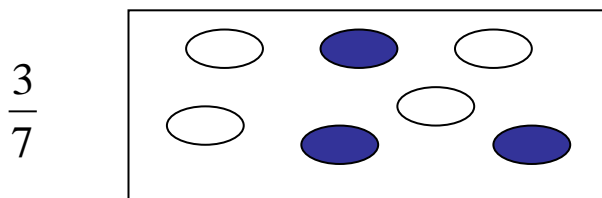
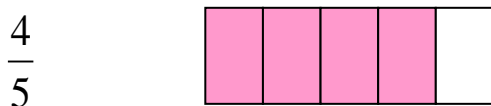
When we talk about a fraction we are saying that it is a part of a whole. An understanding of fractions and how to add, subtract, multiply and divide is a foundation for work in primary mathematics. Explanations of some of the terms that have been used in this document can be found in the [glossary](#) on our website. [Part 2](#) of this document contains exercises to practise

### This sheet will teach you to:

- Recognise different types of fractions 2
- Convert between different types of fractions 2
- Recognise equivalent fractions 3
- Add and subtract fractions 3
- Simplify fractions 5
- Multiply fractions 6
- Divide fractions 7

## 1. Definition of a fraction

A fraction is part of a whole. The fraction  $\frac{4}{5}$  can be demonstrated by shading 4 portions of a whole that has been split into 5 parts.



The fraction bar such as in  $\frac{2}{5}$  means the same as  $2 \div 5$ .

#### Note

Numerator is the top number in the fraction

Denominator is the bottom number in the fraction

#### Note

Depending on your calculator your fraction button could look like  $a^b/c$  or  $\frac{\blacksquare}{\blacksquare}$

## 2. Types of fractions

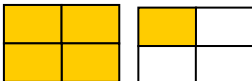
There are three types of fractions:

- a **proper fraction** has a smaller numerator than the denominator so represents a portion less than one whole: for instance  $\frac{1}{2}$
- an **improper fraction** has a numerator bigger than the denominator so represents a portion more than one whole: for instance  $\frac{11}{3}$
- a **mixed numeral** has a whole number as well as a fraction part for instance  $2\frac{1}{3}$

## 3. Converting between different types of fractions

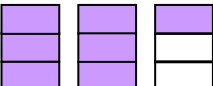
### Converting a mixed numeral to an improper fraction

Examples

a)  $1\frac{1}{4}$  

as the denominator here is 4 then one whole represents  $\frac{4}{4}$

$$\text{therefore } 1\frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

b)  $2\frac{1}{3}$  

as the denominator here is 3 then one whole represents  $\frac{3}{3}$

$$\text{therefore } 2\frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$$

#### To calculate the numerator

When converting from a mixed to improper fraction:

- Multiply the whole number of the mixed fraction by the denominator
- Add on the numerator of the fraction part

c)  $2\frac{1}{6} = \frac{6}{6} + \frac{6}{6} + \frac{1}{6} = \frac{13}{6}$

d)  $3\frac{4}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{4}{5} = \frac{19}{5}$

### Converting an improper fraction to a mixed numeral

Examples

a)  $\frac{14}{4}$  1 whole represents  $\frac{4}{4}$  so there are  $\frac{4}{4}$  &  $\frac{4}{4}$  &  $\frac{4}{4}$  &  $\frac{2}{4}$  left over

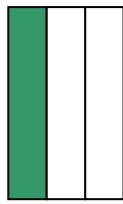
Or  $\begin{array}{r} 3r2 \\ 4 \overline{)14} \end{array}$  so  $\frac{14}{4} = 3\frac{2}{4}$

b)  $\frac{8}{5}$  divide 8 by 5.  $\begin{array}{r} 1r3 \\ 5 \overline{)8} \end{array}$

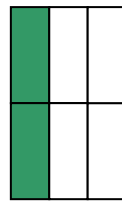
The result is 1 and 3 left over so  $\frac{8}{5} = 1\frac{3}{5}$

## 4. Equivalent fractions

Two fractions that are equivalent represent exactly the same portion of the whole.



$$\frac{1}{3}$$



$$\frac{2}{6}$$

This process either breaks the whole up into smaller portions or expresses the fraction in the simplest or most basic form.

If you multiply both the numerator and denominator of a fraction by a number then you chop the basic fraction up into smaller portions.

### Examples

a)  $\frac{1^{\times 2}}{3_{\times 2}} = \frac{2}{6}$  (as shown above)

b)  $\frac{1^{\times 10}}{3_{\times 10}} = \frac{10}{30}$

### Note

When working with fractions remember that what you do to the top you must do to the bottom

The value of the fraction is unchanged because you are multiplying by the equivalent of 1.

$$\left(\frac{2}{2} \text{ or } \frac{10}{10}\right).$$

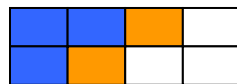
Equivalent fractions are often formed when we are adding and subtracting fractions.

## 5. Adding and subtracting fractions

Adding and subtracting fractions is a process of collecting together like fractions. If the denominators are the same (like fractions) then just add or subtract the numerators.

### Examples

a)  $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

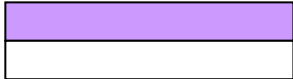
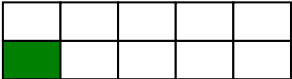


b)  $\frac{7}{11} - \frac{3}{11} = \frac{4}{11}$



When the denominators are different you must form equivalent fractions to make the denominators the same.

**Examples**

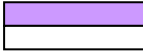
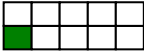
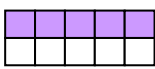
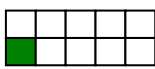

c)  $\frac{1}{2} + \frac{1}{10}$   + 

First find the common multiples of the denominators. This will let us find the lowest common denominator (LCD)

2: 2, 4, 6, 8, **10**, .....

10: **10**, 20, 30, .... 10 is the LCD - convert both fractions to 10ths :

We can look at the steps that need to be taken through a table

$\frac{1}{2} + \frac{1}{10}$	LCD = 10, both denominators become 10	 
$= \frac{1^{\times 5}}{2^{\times 5}} + \frac{1}{10}$	to make $\frac{1}{2}$ into $\frac{?}{10}$ multiply top and bottom by 5	 
$= \frac{5}{10} + \frac{1}{10}$	now the fractions are like so add the numerators	
$= \frac{6}{10}$		


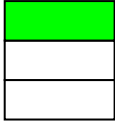
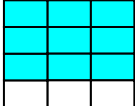

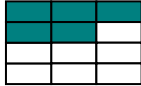
d)  $\frac{3}{4} - \frac{1}{3}$  multiples 4: 4, 8, **12**, 16, .....

3: 3, 6, 9, **12**, ...

12 is the LCD - convert both fractions to 12ths

to make  $\frac{3}{4}$  into  $\frac{?}{12}$  multiply top and bottom by 3

to make  $\frac{1}{3}$  into  $\frac{?}{12}$  multiply top and bottom by 4

$\frac{3}{4} - \frac{1}{3}$	LCD = 12	 
$= \frac{3^{\times 3}}{4^{\times 3}} - \frac{1^{\times 4}}{3^{\times 4}}$	Form equivalent fractions to make each denominator = 12	
$= \frac{9}{12} - \frac{4}{12}$	As soon as the fractions are like fractions subtract the numerators	 
$= \frac{5}{12}$		

e)  $\frac{8}{9} - \frac{5}{6}$       LCD = 18

So

$$\begin{aligned} & \frac{8}{9} - \frac{5}{6} \\ &= \frac{8^{\times 2}}{9^{\times 2}} - \frac{5^{\times 3}}{6^{\times 3}} \\ &= \frac{16}{18} - \frac{15}{18} \\ &= \frac{1}{18} \end{aligned}$$

## 6. Simplifying fractions

When working out a fractions question, the answer should be presented as a proper fraction expressed in its simplest form or as a mixed numeral expressed in its simplest form.

To simplify a fraction divide the numerator and denominator by a common factor.

### Examples

- a) For part a in section 6, the answer of  $\frac{6}{10}$  can be simplified as follows:

$$\frac{6}{10} \quad \text{2 is a factor of 6 and 10 so we divide top and bottom by 2}$$

$$\frac{6^{\div 2}}{10^{\div 2}} = \frac{3}{5}$$

- b)  $\frac{20}{25}$       5 is a factor of 20 and 25 so we divide top and bottom by 5

$$\frac{20^{\div 5}}{25^{\div 5}} = \frac{4}{5}$$

- c)  $\frac{80}{140}$       10 is a factor of 80 and 140 so we divide top and bottom by 10

$$\frac{80^{\div 10}}{140^{\div 10}} = \frac{8}{14}$$

OR  $\frac{80}{140} = \frac{8}{14}$

Then we look at the factors again 2 is a factor of 8 and 14

$$\frac{8^{\div 2}}{14^{\div 2}} = \frac{4}{7}$$

### Note

A common factor is a number that divides evenly into both the numerator and denominator.

Again this is equivalent to dividing by 1 so will not change the value of the fraction.

### Note

Usually when 10 or a power of 10 is the common factor we cancel out the zeros on the top and bottom

$$d) \frac{36}{16} = \frac{36^{\div 4}}{16^{\div 4}} = \frac{9}{4}$$

as this answer is an improper fraction, we can convert it to a mixed numeral.

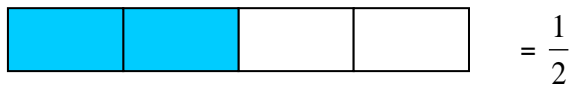
$$\frac{9}{4} = 2\frac{1}{4}$$

## 7. Multiplying fractions

Multiplying fractions is a way of chopping the fraction up.

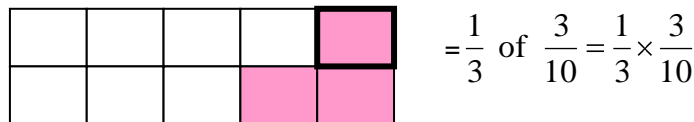
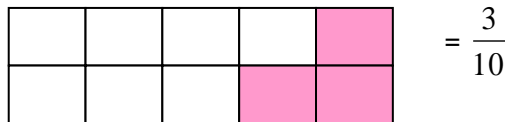
### Examples

$$a) \frac{1}{2} \times \frac{1}{2} \text{ means find } \frac{1}{2} \text{ of } \frac{1}{2}$$



$$\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

$$b) \frac{1}{3} \times \frac{3}{10} \text{ means find } \frac{1}{3} \text{ of } \frac{3}{10}$$



$$\frac{1}{3} \times \frac{3}{10} = \frac{1 \times 3}{3 \times 10} = \frac{3^{\div 3}}{30^{\div 3}} = \frac{1}{10}$$

$$\text{Or } \frac{1}{3} \times \frac{3}{10} = \frac{1 \times 3^{\div 3=1}}{3^{\div 3=1} \times 10} = \frac{1 \times 1}{1 \times 10} = \frac{1}{10}$$

#### Note

The rule when multiplying fractions is to multiply the numerators and multiply the denominators. Then simplify the answer if possible.

#### Note

An alternative method here is to cancel or simplify before multiplying out.

$$c) 4 \times \frac{5}{7}$$

$$4 \times \frac{5}{7} = \frac{4}{1} \times \frac{5}{7} = \frac{4 \times 5}{1 \times 7} = \frac{20}{7} = 2\frac{6}{7}$$

#### Note

Any whole number can be written as a fraction by using 1 as the denominator.

$$d) \frac{3}{8} \times 20 = \frac{3}{8} \times \frac{20}{1} = \frac{3 \times 20}{8 \times 1} = \frac{60^{\div 2}}{8_{\div 2}} = \frac{30^{\div 2}}{4_{\div 2}} = \frac{15}{2} = 7\frac{1}{2}$$

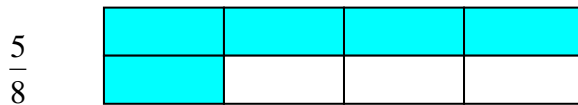
$$\text{or } \frac{3}{8} \times \frac{20}{1} = \frac{3 \times 20^{\div 4=5}}{8_{\div 4=2} \times 1} = \frac{3 \times 5}{2 \times 1} = \frac{15}{2} = 7\frac{1}{2}$$

## 8. Dividing fractions

When dividing fractions you are asking how many shares you have.

### Examples

a)  $\frac{5}{8} \div \frac{1}{4}$  is asking how many one quarter shares there are in  $\frac{5}{8}$



**Asking**

How many  $\frac{1}{4}$ 's in  $\frac{5}{8}$  ???.

Each  $\frac{2}{8}$ ths represents a one quarter share, so  $\frac{5}{8}$  ths represents just over 2 shares

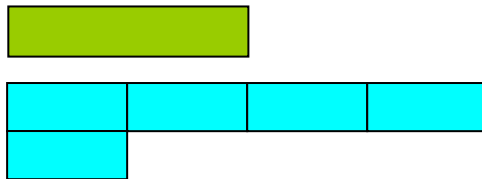
$$\frac{5}{8} \div \frac{1}{4} = \frac{5}{8} \times \frac{4}{1} = \frac{5 \times 4}{8 \times 1} = \frac{20^{\div 4}}{8_{\div 4}} = \frac{5}{2} = 2\frac{1}{2}$$

$\div \frac{1}{4}$  is the same as  $\times 4$

**Rule**

Invert the second fraction.  
Multiply the two fractions.  
Then simplify.

b)  $\frac{1}{4} \div \frac{5}{8}$  is asking how many five eights shares there are in  $\frac{1}{4}$



$\frac{5}{8}$  is a bigger fraction than  $\frac{1}{4}$  so the answer will be less than one.

$$\frac{1}{4} \div \frac{5}{8} = \frac{1}{4} \times \frac{8}{5} = \frac{1 \times 8}{4 \times 5} = \frac{8^{\div 4}}{20_{\div 4}} = \frac{2}{5}$$

**Note**

To multiply and divide mixed numerals you must first convert each fraction to its improper form or use a calculator.

c)  $\frac{3}{11} \div 4 = \frac{3}{11} \div \frac{4}{1} = \frac{3}{11} \times \frac{1}{4} = \frac{3 \times 1}{11 \times 4} = \frac{3}{44}$

d)  $5 \div \frac{6}{8} = \frac{5}{1} \div \frac{6}{8} = \frac{5}{1} \times \frac{8}{6} = \frac{5 \times 8^{\div 2=4}}{1 \times 6_{\div 2=3}} = \frac{5 \times 4}{1 \times 3} = \frac{20}{3} = 6\frac{2}{3}$

## 9. For more information

Visit our Learning Skills website at <http://www.csu.edu.au/division/studserv/maths/teachered.htm>

Part 2 of this document for exercises relating to this topic can be found at <http://www.csu.edu.au/division/studserv/maths/teachered.htm>

Other useful websites are available at:

<http://www.coolmath.com/fractions/index.html>

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