



The Independent Measures t statistic

Learning Skills

Introduction:

When comparing two small sample groups whose means and standard deviations are known, an independent measures t statistic will be calculated

This sheet will teach you to:

- Understand the mathematical notation
- Work through the formula step by step
- Interpret the result in terms of the similarity or otherwise of the two groups

1. The formula

Example

Find the t score for the following two samples

Scores for group 1: 9, 5, 9, 8, 1, 2, 6, 4, 2, 10, 7, 6, 9

Scores for group 2: 4, 12, 11, 5, 10, 13, 9, 8, 5, 11, 10, 7, 6, 15

The formula is

$$t = \frac{M_1 - M_2}{\sqrt{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)}}$$

Where $s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}$

Symbols

- M_1 and M_2 are the means of group 1 and group 2
- S_p^2 is the pooled variance
- SS_1 and SS_2 are the sum of squares for each group
- n_1 and n_2 are the number of scores in each group
- df_1 and df_2 are the degrees of freedom for each group
- $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$

2. Calculating means, sum of squares and degrees of freedom

For group 1:

$$\Sigma X = 9+5+9+8+1+2+6+4+2+10+7+6+9 = 78$$

$$(\Sigma X)^2 = 78^2 = 6084$$

$$\Sigma X^2 = 9^2+5^2+9^2+8^2+1^2+2^2+6^2+4^2+2^2+10^2+7^2+6^2+9^2 = 578$$

$$n_1 = 13, \quad M_1 = \frac{\sum X}{n} = \frac{78}{13} = 6 \quad df_1 = 13 - 1 = 12$$

$$SS_1 = \sum X^2 - \frac{(\sum X)^2}{n} = 578 - \frac{6084}{13} = 517 - 468 = 49$$

For group 2:

$$\sum X = 4+12+11+5+10+13+9+8+5+11+10+7+ 6+15 = 126$$

$$(\sum X)^2 = 126^2 = 15\,876$$

$$\sum X^2 = 4^2+12^2+11^2+5^2+10^2+13^2+9^2+8^2+5^2+11^2+10^2+7^2+6^2 +15^2 = 1276$$

$$n_2 = 14 \quad M_2 = \frac{\sum X}{n} = \frac{126}{14} = 9 \quad df_2 = 14 - 1 = 13$$

$$SS_2 = \sum X^2 - \frac{(\sum X)^2}{n} = 1276 - \frac{15876}{14} = 1276 - 1134 = 142$$

3. Calculate the pooled variance

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2} = \frac{49 + 142}{12 + 13} = \frac{191}{25} = 7.64$$

4. Calculate the t statistic

$$t = \frac{M_1 - M_2}{\sqrt{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)}} = \frac{6 - 9}{\sqrt{\left(\frac{7.64}{13} + \frac{7.64}{14}\right)}} = \frac{-3}{\sqrt{(0.588 + 0.546)}} = \frac{-3}{\sqrt{1.134}} = \frac{-3}{1.06} = -2.83$$

5. How to interpret this t statistic

The t statistic is negative because group 2's mean is bigger than group 1's mean.

This t statistic of 2.83 means there is quite a difference between the two means. If the means were the same then the t statistic would equal 0. The bigger the t statistic the stronger the evidence that there is a statistically significant difference between the two means. A process called a hypothesis test could be conducted in order to test for significance.

6. For more information

Visit our Learning Skills website at <http://www.csu.edu.au/division/studserv/math/index.htm>

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