



# Solving a quadratic equation

## Learning Skills

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### Introduction:

*Quadratic equations that can be solved yield either 1 or 2 solutions for the variable.*

### This sheet will teach you to:

- Identify a solvable quadratic equation
- Solve a quadratic equation

## 1. What is a quadratic equation

A quadratic equation is a polynomial equation for which the highest power (sometimes called the degree) is 2.

The general form is  $y = ax^2 + bx + c$  where a, b and c are numbers.

Examples of quadratic equations are  $y = 2x^2 + 5x - 3$  and  $y = x^2 - 7$ .

If y is set at 0 as in  $2x^2 + 5x - 3 = 0$  or  $x^2 - 7 = 0$ , then the quadratic can be solved to find values for x that make the equation true.

## 2. Solving a quadratic equation

To solve these equations a formula can be used to find the 1 or 2 possible solutions for x. The formula will also indicate if there are no solutions.

The formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The a represents the number before the  $x^2$  (called the  $x^2$  coefficient)

The b is the number before the x (called the x coefficient)

And c is the unattached number (called the constant)

### Example

Solve  $2x^2 + 5x - 3 = 0$

$a = 2$ ,  $b = 5$  and  $c = -3$  (don't forget to include the sign as well)

Substituting into the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - (4 \times 2 \times -3)}}{2 \times 2}$$

$$x = \frac{-5 \pm \sqrt{25 - (-24)}}{4}$$

$$x = \frac{-5 \pm \sqrt{49}}{4}$$

$$x = \frac{-5 + 7}{4} \text{ or } \frac{-5 - 7}{4}$$

$$x = 0.5 \text{ or } -3$$

#### Note

The two possible solutions for  $x$  come about with the  $\pm$  operation. This requires adding the following term as well as subtracting the following term as in line 5.

There are 2 possible solutions for  $x$ : either  $x = -3$  or  $x = 0.5$ . These solutions can be called the roots of the quadratic.

Check: for  $x = -3$ ; left hand side  $= 2 \times (-3)^2 + 5 \times (-3) - 3 = 2 \times 9 - 15 - 3 = 18 - 15 - 3 = 0$

right hand side  $= 0$

Left hand side  $=$  right hand side so  $x = -3$  is a solution

The solution  $x = 0.5$  can be checked similarly

### Example

Solve  $x^2 - 7$

$a = 1$ ,  $b = 0$  (there is no  $x$  term),  $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{0 \pm \sqrt{0 - (4 \times 1 \times -7)}}{2 \times 1}$$

$$x = \frac{0 \pm \sqrt{0 - (-28)}}{2}$$

$$x = \frac{0 \pm \sqrt{28}}{2}$$

$$x = \frac{0 + 5.292}{2} \text{ or } \frac{0 - 5.292}{2}$$

$$x = 2.646 \text{ or } -2.646$$

#### Note

28 does not have an exact square root so the two solutions are approximate answers to 3 decimal places.

When working through the formula, if the number under the square root is negative, then the equation has no solutions as you cannot find the square root of a negative number.

#### Example

Solve  $2x^2 + x + 10 = 0$

$a = 2, b = 1, c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - (4 \times 2 \times 10)}}{2 \times 2}$$

$$x = \frac{-1 \pm \sqrt{1 - (80)}}{4}$$

$$x = \frac{0 \pm \sqrt{-78}}{2}$$

As  $\sqrt{-78}$  does not exist,  $2x^2 + x + 10 = 0$  has no solutions

### 3. For more information

Visit our Learning Skills website at <http://www.csu.edu.au/division/studserv/maths/index.htm>

Other useful websites are available at:

<http://www.coolmath.com/algebra/>

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