Boundary shear stress distribution in meandering compound channel flow

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Abstract

Reliable prediction of boundary shear force distributions in open channel flow is crucial in many critical engineering problems such as channel design, calculation of energy losses and sedimentation. During floods, part of the discharge of a river is carried by the simple main channel and the rest is carried by the floodplains located to its sides. For such compound channels, the flow structure becomes complicated due to the transfer of momentum between the deep main channel and the adjoining floodplains that magnificently affects the shear stress distribution in floodplain and main channel sub sections. Knowledge of momentum transfer at the different interfaces originating from the junction between main channel and floodplain can be acquired from the distribution of boundary shear in the sub sections. An investigation concerning the distribution of shear stress in the main channel and floodplain of meandering and straight compound channels are presented. Based on the experimental results of boundary shear, this paper predicts the distribution of boundary shear carried by main channel and floodplain sub sections. Five dimensionless parameters are used to form equations representing the total shear force percentage carried by floodplains. A set of smooth and rough sections is studied with aspect ratio varying from 2 to 5. The models are also validated using the data of other investigators.

Keywords

Boundary shear, aspect ratio, interface plane, compound section, meander, apparent shear

Introduction

Information regarding the nature of boundary shear stress distribution in flowing simple and compound channels are needed to solve a variety of river hydraulics and engineering problems such as to give a basic understanding of the resistance relationship, to understand the mechanism of sediment transport and to design stable channels etc. The boundary shear stress distribution, velocity distribution and flow resistance in compound cross section channels have been investigated by a number of authors (Wright and Carstens, 1970, Ghosh and Jena, 1971, Myers and Elsayw, 1975, Rhodes and Knight, 1994, Patra, 1999, Patra and Kar, 2000). Most hydraulic formulae assume that the boundary shear stress distribution is uniform over the wetted perimeter. Distribution of boundary shear stress mainly depends upon the shape of the cross section and the structure of the secondary flow cells. However, for meander channel - floodplain geometry, there is wide variation in the local shear stress distribution from point to point in the wetted perimeter. Also the magnitude of boundary shear of a meandering channel is significantly different from that of straight channel having the same geometry, shape and cross sectional area. Therefore, there is a need to evaluate the boundary shear stress carried by the main channel and floodplain walls at various locations of meander path. The aim of this study is to describe the effect of the interaction mechanism on the basis of shear stress distribution in meandering and straight compound channel sections.

Methods

Schematic diagram showing experimental set up from three types of channels along with the plan forms of the meandering channels with floodplains are shown in Fig.1. The summary of experimental runs for the compound channel geometries are given in Table 1. Type I channel is asymmetrical with two unequal floodplains attached to both sides of the main channel. Type-II and IIR channels are asymmetrical with only floodplain attached to one side of the main channel. All surfaces of the channel IIR are roughened with rubber beads of 4 mm diameter at 12 mm centre to centre. Type-III channel is symmetrical with two equal floodplains attached to both sides of the main channel. A recirculating system of water supply is established with pumping of water from an
underground sump to an overhead tank from where water could flow under gravity to a stilling tank. From the stilling tank water is led to the experimental channel through a baffle wall. A transition zone helped to reduce turbulence of the water flow. An adjustable tailgate at the downstream end of the flume is used to achieve uniform flow over the test reach. Water from the channel is collected in a measuring tank. The measuring devices consist of a point gauge mounted on a traveling bridge to measure depths having a least count of 0.1 mm. A Preston micro-Pitot tube in conjunction with a water manometer is used to measure dynamic pressures for the evaluation of boundary shear stress and velocity.

![Fig. 1 Schematic Diagram showing Experimental Setup (Plan View)](image)

The ratio $\alpha$ between overall width $B$ and main channel width $b$ of the meandering compound channels could be varied from 2.13 to 5.25 for the three types of channels. The compound channel sections are made from Perspex sheets and observations were made at the section of maximum curvatures (bend apex) of the meandering channel geometries. For each channel, boundary shear stress measurements covering a number of points in the wetted perimeter for each location have been obtained from the dynamic pressure drop measured by Preston tube - micro manometer system and from the semi log plot of velocity distribution. The diameter of the Preston tube used is such that it lies in the region of dynamic similarity, which is about one-fifth of the boundary layer thickness. While dealing with the flow over rough boundaries care has been taken to locate the tube from zero datum such that the roughness distribution does not influence the recording of the dynamic pressure drop. In view of the complexities involved in converting the dynamic pressure drop to shear stress, the computed mean shear by both the approaches are compared with the energy gradient approach and the closest values of shear distribution are considered.

![Table 1. Summary of experimental runs for meandering channel with floodplains.](image)

**Table 1. Summary of experimental runs for meandering channel with floodplains.**

<table>
<thead>
<tr>
<th>Experiment series/Run No</th>
<th>Nature of Channel surface</th>
<th>Bed slope</th>
<th>Top width $B$ (cm)</th>
<th>Main channel width $b$ (cm)</th>
<th>Total depth of Flow $H$ (cm)</th>
<th>Depth of lower main channel $h$ (cm)</th>
<th>Simposity $S_r$</th>
<th>Amplitude Width ratio ($R$)</th>
<th>% age of flood plain shear ($% S_f$)</th>
<th>Observed Discharge $Q$ (cm$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1,A.2,A.3 smooth</td>
<td>0.0064</td>
<td>52.5</td>
<td>10</td>
<td>11.6,14.9,16.8</td>
<td>10</td>
<td>1.22</td>
<td>0.178</td>
<td>64.1, 67.0, 67.4</td>
<td>3960,14000,19500</td>
<td></td>
</tr>
<tr>
<td>C.4,C.5,C.6 smooth</td>
<td>0.004</td>
<td>21.3</td>
<td>10</td>
<td>12.19,13.81,15.2</td>
<td>10</td>
<td>1.21</td>
<td>(-)0.481</td>
<td>29.0,34.8,38.0</td>
<td>5800,8450,11250</td>
<td></td>
</tr>
<tr>
<td>D.7,D.8 smooth</td>
<td>0.004</td>
<td>41.8</td>
<td>10</td>
<td>12.19,14.08</td>
<td>10</td>
<td>1.21</td>
<td>0.245</td>
<td>59.2,59.9</td>
<td>5800,8450</td>
<td></td>
</tr>
<tr>
<td>F.9,F.10,F.11 rough</td>
<td>0.004</td>
<td>21.3</td>
<td>10</td>
<td>12.22,13.71,15.23</td>
<td>10</td>
<td>1.21</td>
<td>(-)0.481</td>
<td>27.1,34.4,36.1</td>
<td>5500,8200,10900</td>
<td></td>
</tr>
<tr>
<td>G.12,G.13,G.14 rough</td>
<td>0.004</td>
<td>41.8</td>
<td>10</td>
<td>12.49,14.23,15.84</td>
<td>10</td>
<td>1.21</td>
<td>0.245</td>
<td>55.9,56.0</td>
<td>5500,8200,10900</td>
<td></td>
</tr>
<tr>
<td>I.15, I.16, I.17 smooth</td>
<td>0.00278</td>
<td>138</td>
<td>44</td>
<td>29.5,30.7,31.6</td>
<td>25</td>
<td>1.043</td>
<td>0.072</td>
<td>37.1,42.8</td>
<td>94535,103537,108583</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 2(a) Isovel and Boundary shear stress distribution Of Meandering Channel with both side Flood plain**

**Fig. 2(b) Isovel and Boundary shear stress distribution Of Meandering Channel with one side Flood plain**

**Results**

**Boundary shear force results**

The shear stress distribution for two typical channels (Table 1) A-3 and C-6 are shown in Fig. 2 (a) and Fig. 2 (b) respectively. Various boundary elements of the compound channels are labeled from 1– 4 in Fig. 3. Label (1) denotes the vertical wall(s) of floodplain with length \(2(H-h)\) and label (2) denotes floodplain bed(s) with length \((B-b)\). Label (3) denotes the two main channel walls and the bed of the main channel is represented by label (4). The measured shear stresses are integrated over the respective lengths of each boundary elements to obtain the boundary shear force per unit length for the elements. The total shear force carried by the floodplain beds and walls are very important because the apparent shear force acting on the assumed interfaces originating from the junction between floodplain and main channel can be determined once this quantity is known.

The sum of the boundary shear forces for all beds and walls of the compound channel is used as a divisor to calculate the shear force percentages carried by the boundary elements. The shear force percentage carried by the floodplains is represented as \(\%S_f\), and for the main channel it is represented as \(\%S_m\). Shear force percentages carried by the floodplain \(\%S_f\) with depth ratio \((H-h)/H\) for all of the compound channels for varying geometries \((\alpha = 2.13–5.25)\) are given in Table 1. It can be seen from Table 1 that the value of floodplain shear increases with depth of flow in the compound channel. For the meandering and straight compound channels, the variation of shear force in floodplains with relative depth \(\beta = (H-h)/H\) for different values of \(\alpha\) \((=B/b)\) are shown in Fig. 4. The percentage of total shear force carried by the floodplain beds and walls \(\%S_f\) for compound channel increases with increase in, relative depth \(\beta\), the channel width ratio \(\alpha\) and sinuosity \(S_r\).

![Figure 3. Notations](image)

**Figure 3. Notations**

The apparent shear force on various interfaces

The apparent shear force at the assumed interface plane gives an insight into the magnitude of flow interaction between the main channel and the adjacent floodplains basing on which the merits of the selection of the interface planes for discharge estimation are decided. The conventional method of discharge calculation in compound sections divides the channel into hydraulically homogeneous regions by plane originating from the junction of the floodplain and main channel, so that the floodplain region can be considered as moving separately from the main channel. The assumed plane may be: (1) Vertical interface aa1; (2) horizontal interface aa or (3) diagonal interface aa2 (Fig.3). Once the shear force carried by the floodplain is known, the apparent shear force acting on the imaginary interface of the compound section can be calculated. These apparent shear forces may then be used to get an idea of the momentum transfer between the different subsections of the compound channel. For any regular prismatic channel under uniform flow conditions the sum of boundary shear forces acting on the main channel wall and bed together with an “apparent shear force” acting on the interface

![Figure 4a b. Variation of floodplain shear with depth ratio \(\beta\) of straight & meandering compound channel](image)

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plane between main channel and floodplain must be equal to the resolved weight force along the main channel given as

\[ \rho A_{mc} S = \int_{mc} \tau dp + ASF_{ip} \]  

(1)

in which \( g \) = gravitational acceleration, \( \rho \) = density of flowing fluid, \( S \) = slope of the energy line, \( A_{mc} \) = area of the main channel defined by the interface plane, \( \int_{mc} \tau dp \) = shear force on the surfaces of the main channel consisting of two vertical walls and bed, and \( ASF_{ip} \) = apparent shear force of the imaginary interface plane. Because the boundary shear stress carried by the compound section \((\rho g AS)\) is equal to 100%, where \( A \) is the total cross section of the compound channel, the percentage shear force carried by the main channel surfaces can be calculated as

\[ \% S_{mc} = 100 \left[ \frac{\int_{mc} \tau dp}{\rho g A S} \right] = 100 \left[ \frac{\rho A_{mc} S}{\rho g A S} \right] - 100 \frac{ASF_{ip}}{\rho g A S} \] 

(2)

But since \( \% S_{mc} = 100 - \% S_{fp} \); and \( 100(ASF_{ip}/\rho g AS) = \) percentage of shear force on the assumed interface, substituting the values the apparent shear force on the interface plane is calculated as

\[ \% ASF_{ip} = 100 \left[ \frac{A_{mc}}{A} \right] - \left[ 100 - \% S_{fp} \right] \] 

(3)

in which \( \% ASF_{ip} \) = percentage of shear force in the interface plane. These apparent shear forces can be expressed as percentages of the total channel shear force using the following relations:

\[ \% ASF_V = \frac{50}{[\alpha - 1] \beta + 1} - \frac{1}{2} \left[ 100 - \% S_{fp} \right] \] 

(4a)

\[ \% ASF_D = \frac{25}{[\alpha - 1] \beta + 1} - \frac{1}{2} \left[ 100 - \% S_{fp} \right] \] 

(4b)

\[ \% ASF_H = \frac{100}{[\alpha - 1] \beta + 1} - \left[ 100 - \% S_{fp} \right] \] 

(4c)

Percentages of apparent shear force for the assumed vertical, horizontal, and diagonal interface planes may be calculated using (4). Selection of suitable interfaces for calculating conveyance by divided channel method becomes easier once we know the apparent shear in different assumed interfaces. Toebes and Sooky (1967) carried out laboratory experiments on two stage meandering composite channel sections and showed that a nearly horizontal fluid boundary located at the junction between the main channel and flood plain would be more realistic than other interfaces. Wormleaton, et al., (1980, 1982) have proposed an apparent shear stress ratio from the apparent shear by which suitability of interface plain for calculation of discharge can be predicted. Patra and Kar (2000) have proposed the variable interface method of discharge calculation for meandering and straight compound channel. Stephenson and Kolovpoulos (1990) proposed the area method for discharge calculation in compound channel by selecting a curved interface by assuming the apparent shear along the interface length as zero. For the assumed vertical interface plane the shear force is always positive for the ranges of \( \alpha \) and \( \beta \) tested. A positive value indicates transfer of momentum from the main channel to the floodplain at the assumed plane indicating the floodplain flow retarding the main channel flow. This apparent shear stress is higher than the bed-shear stress at low floodplain depths and reduces gradually as \( \beta \) increases. For the diagonal and horizontal interface planes it is observed that the apparent shear force is positive at low depths and changes sign as depth increases indicating that at higher depths over floodplain there is transfer of momentum from the floodplain to the main channel. A smaller value of apparent shear stress renders the interface plane more suitable, but a large negative value of apparent shear stress at higher depths makes the interface plane unsuitable for separating the channel into hydraulically homogeneous zones for calculating discharge of compound channels by the divided channel method.

**Development of the model**

Knight and Hamed (1984) investigated smooth compound channels having a bank full depth of 76 mm that can be considered close to the present channels for which most of data are presented and proposed equation for the percentage of total shear force carried by the floodplain as

\[ \% S_{fp} = 48(\alpha - 0.8)^{0.289}(2\beta)^m \] 

(5a)

where the exponent \( m \) can be calculated from the relation \( m = 1/\left[ 0.75 e^{0.38 \alpha} \right] \). These equations apply to the straight compound channels having symmetry with respect to the main channel centerline. Patra and Kar (2000)
proposed equation for calculating the percentage of total shear force carried by the floodplain for the meandering compound channels as

\[
\%S_{fp} = 48(\alpha - 0.8)^{0.289}(2\beta)^{0.5}[1 + \alpha \text{Re}^{-13.25/63}]^{1 + 1.02(\beta \log_{10} \gamma)}
\]  

(5b)

in which \( R \) = ratio of the amplitude of the meandering channel to the top width \( B \) of the compound section, the values of which are given in col. 9 of Table 1, and \( \delta \) = aspect ratio of the main channel \( b/h \). The equation is valid for the meandering compound sections for smooth boundaries or with same roughness in the floodplain and in the main channel. For the compound channel with different roughness Patra and Kar (2000) further proposed the following general equation for the percentage of floodplain shear as

\[
\%S_{fp} = 48(\alpha - 0.8)^{0.289}(2\beta)^{0.5}[1 + \alpha \text{Re}^{-13.25/63}]^{1 + 1.02(\beta \log_{10} \gamma)}
\]  

(5c)

where \( \gamma \) = the ratio of Manning’s roughness \( n \) of the floodplain to that for the main channel. For straight channel the value of \( R \) is zero. A zero value of \( R \) reduces (5c) to the form of (5b) and for channels with equal surface roughness in the floodplain and main channel for \( \gamma = 1 \), the equation further reduces to the form (5a) proposed by Knight and Hamed (1984). Due to complexity of the empirical equations proposed by the previous investigators a regression analysis is made to obtain a simple but more reliable equation to predict the percentage of floodplain shear. From these plots between \( \%S_{fp} \) versus \( \%S_{fp} \) for the straight compound channel Fig. 5(a) of Knight and Demetriou (1983) and for the meandering compound channel of Patra and Kar (2000) Fig. 5(b) the best fitted simple linear function is obtained and for a straight compound channel the equation for percentage of shear carried by floodplain is modeled as

\[
\%S_{fp} = 0.822 A_{f} + 19.58 \quad \text{or} \quad \%S_{fp} = 0.822 \frac{\beta (\alpha - 1)}{1 + \beta (\alpha - 1)} + 19.58
\]  

(6)

Figure 5a. &b. Variation of % of shear in Floodplain perimeter with that of area in floodplain.

Equation (6) is valid for straight compound channel with smooth surfaces only. For different roughness in main channel and floodplain surface, Equation (6) is further improved and is represented as

\[
\%S_{fp} = \left[ 0.822 \frac{\beta (\alpha - 1)}{1 + \beta (\alpha - 1)} + 19.58 \right] 1 + 1.02(\beta \log_{10} \gamma)
\]  

(7)

where \( \gamma = \) the ratio of Manning’s roughness \( n \) of the floodplain and that for the main channel. For meandering channel with floodplain the distribution is further complicated and modified due to meandering effect. It has been observed from the experimental result that the percentage of boundary shear is exponentially on amplitude/width ratio \( R \). Finally a general model to represent the percentage of total shear force carried by floodplain of meandering compound channel is given as

\[
\%S_{fp} = \left[ 0.822 \frac{\beta (\alpha - 1)}{1 + \beta (\alpha - 1)} + 19.58 \right] 1 + 1.02(\beta \log_{10} \gamma)
\]  

(8)

Reviews of literature show that investigators propose alternatives interface planes to calculate the total discharge carried by a compound channel section. Either including or excluding the interface length does not make sufficient allowance for discharge calculation for all depths of flow over floodplain. It either overestimates or underestimates the discharge results due to neglect of momentum transfer in terms of apparent shear at the respective interfaces. Having computed \( \%S_{fp} \) through (8), it is easy to evaluate (4) for the assumed interface plane. After finding out apparent shear in the assumed interface of a compound channel, it becomes easier for selection of suitable interfaces for discharge calculation. While calculating discharge of compound channel using divided channel method if the apparent shear is found to be negligible for any interface for a particular depth of flow, than the interface length is not included for discharge calculation using Manning’s equation. The total section discharge is obtained by adding all the sub section discharges. Again for a depth of flow in a compound channel, if apparent shear is found to be equal to boundary shear than the interface lengths
is added to the subsection perimeter of main channel to obtain the correct discharge. If apparent shear is very large compared to boundary shear of the subsection perimeter for any over bank depth of flow than the selection of respective interface plain gives erroneous discharge result by using the divided channel method. Therefore simply including/excluding the interface lengths to the subsection wetted perimeters for discharge calculation do not justify the amount of momentum transfer in the respective interfaces of the over bank flow. The variation of computed percentage of shear force of floodplain wetted perimeter with the observed value for meandering compound channels of Patra and Kar (2000) is plotted in Fig. 6(a) while that for the straight compound channels of Knight and Demetrious (1983) is shown in Fig. 6(b). Fig.6 shows the adequacy of equation (8).

![Figure 6a. & b. Variation of observed value and modeled value of floodplain shear.](image)

**Conclusion**

For the meandering compound channels the important parameters effecting the boundary shear distribution are sinuosity ($S_r$), the amplitude ($\varepsilon$), relative depth ($\beta$) and the width ratio ($\alpha$) and the aspect ratio ($\delta$). These five dimensionless parameters are used to form general equations representing the total shear force percentage carried by floodplains. The proposed equations give good result with the observed data for the straight compound channel of Knight and Demetrious (1983) as well as for the meandering compound channels. The model is adequate in defining the relationship for $\% S_f$. It is recommended that further investigation be focused on extending the present analysis to the compound channel of different cross sections such as trapezoidal cross sections.

**References**


